

# Spectra of heavy-light and heavy-heavy mesons containing charm quarks, including higher spin states for $N_f = 2 + 1$

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We study the spectra of heavy-light and heavy-heavy mesons containing charm quarks, including higher spin states. We use two sets of  $N_f = 2 + 1$  gauge configurations, one set from QCDSF using the SLiNC action, and the other configurations from the Budapest-Marseille-Wuppertal collaboration, using the HEX smeared clover action. To extract information about the excited states, we choose a suitable basis of operators to implement the variational method.

*The XXIX International Symposium on Lattice Field Theory, Lattice2011*

*July 11-16, 2011*

*The Village at Squaw Valley, California, USA*

\*Speaker.

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## 1. Introduction

Charm physics has undergone a renaissance in recent years. The spectroscopy of hadrons containing charm quarks has been given prominence through the discovery of several new narrow charmonium resonances close to the  $D\bar{D}$  thresholds and new narrow  $D_s$  mesons close to the  $DK$  thresholds (for reviews see refs. [1, 2]). These states are likely to require the extension of non-relativistic quark models to include the 4-quark sector. A wealth of results is expected over the next few years from completed and existing experiments, for example BaBar, Belle, BES-III and the LHC experiments, and the future PANDA experiment at the FAIR facility.

Lattice calculations of the spectroscopy of hadrons containing charm quarks are vital for interpreting the experimental results. We present here preliminary results on the lower lying spectra (with angular momentum  $L \leq 3$ ) of charmonia,  $D$  and  $D_s$  mesons. The calculations have been carried out on two different sets of configurations, provided by the BMW-c and QCDSF collaborations, both with  $N_f = 2 + 1$  dynamical flavours.

This write-up is organised as follows: in section 2 we give details of our computational setup and the methods used. Then, we present some preliminary results on the  $D_s$  and charmonium spectra before summarising.

## 2. Computational setup

The two sets of  $N_f = 2 + 1$  configurations provided by BMW-c and QCDSF both were generated using the tree-level Symanzik-improved gluonic action. The BMW-c configurations employ tree-level clover improved Wilson fermions, coupled to links that have undergone two levels of HEX smearing [3]. The QCDSF ensembles use non-perturbatively improved Wilson fermions with stout links in the derivative terms (SLiNC action [4]).

BMW-c has generated ensembles spanning a range of lattice spacings, from  $a \approx 0.054$  fm to  $a \approx 0.092$  fm and pion masses, from  $M_\pi \approx 520$  MeV down to  $M_\pi \approx 120$  MeV, which is below the physical point. The volumes range from  $32^3 \times 64$  to  $64^3 \times 144$ , and the number of measurements per set for the present study is  $\sim 200$ . For more details on the configurations see reference [5]. With a reasonable range of lattice spacings we can perform a controlled continuum limit extrapolation of spectral quantities.

The configuration generation of QCDSF is at an earlier stage. A different approach is taken to varying the sea quark masses: these are chosen by first finding the  $SU(3)_{\text{flavour}}$ -symmetric point where flavour singlet mass averages take their physical values. Subsequently, the individual quark masses are varied while keeping the singlet quark mass  $\bar{m}_q = (m_u + m_d + m_s)/3$  constant [6, 7]. Configurations have been generated at  $\beta = 10/g^2 = 5.5$ , corresponding to  $a \sim 0.08$  fm, at several values of  $m_{u/d}$  and  $m_s$  and two volumes of  $24^4 \times 48$  and  $32^3 \times 64$  lattice points.

In this study we have so far analysed the two ensembles whose details are given in table 1. The approach taken to varying the quark mass means the QCDSF ensembles are ideal for studying flavour symmetry violations, particularly in the  $D/D_s$  spectra. The ensembles chosen are at the symmetric point and the ensemble with the lightest sea quark mass (i.e. the biggest difference between  $m_{u/d}$  and  $m_s$ ).

$\kappa_l$	$\kappa_s$	$a$ fm	# meas	$M_\pi$ (MeV)
0.12090	0.12090	0.0795(3)	941	442
0.12104	0.12062	0.0795(3)	450	348

**Table 1:** Details of the QCDSF configurations used so far in this study.

In order to extract information about the ground and excited states we use the variational method [8, 9], with a basis of three different smearing functions. We choose a sub-set of the bilinear operators for the mesons given in Ref. [10]. For each operator we vary the smearing to enable us to find three smearings which are sufficiently different, so that the ground state and first two excited states could be extracted: this corresponds to a smearing function that is close to optimal for the ground state and two others that contain significant excited state contributions. For the smearing, we used gauge-invariant Wuppertal smearing [11, 12] with APE smeared links [13].

### 3. Results

Figure 1 shows the charmonium (top) and  $D_s$  (bottom) spectra. The filled points correspond to the results obtained from the QCDSF configurations at the flavour symmetric point. We have been able to extract the excited states as well as ground state for all operators. The second excited state is only included if a clear plateau was observed in the effective mass plots. This holds for many of the charmonium states but only for two operators in the  $D_s$  spectrum. Since the charm quark mass has not been tuned precisely yet, we have shifted the results so that the spin average 1S mass agrees with the experimental data. Qualitatively, the experimental spectra are reproduced. We use the simplest identification of the J quantum number with lattice representations, but work is in progress using the methods in [14].

In the  $D_s$  spectrum plot, we also show results obtained on the BMW-c configurations that are extrapolated to the physical light quark mass and continuum limit. The corresponding extrapolation for the  $D_s$  hyperfine splitting is shown in Figure 2. A combined fit encapsulating the lattice spacing and quark mass dependence is carried out using the fit function,

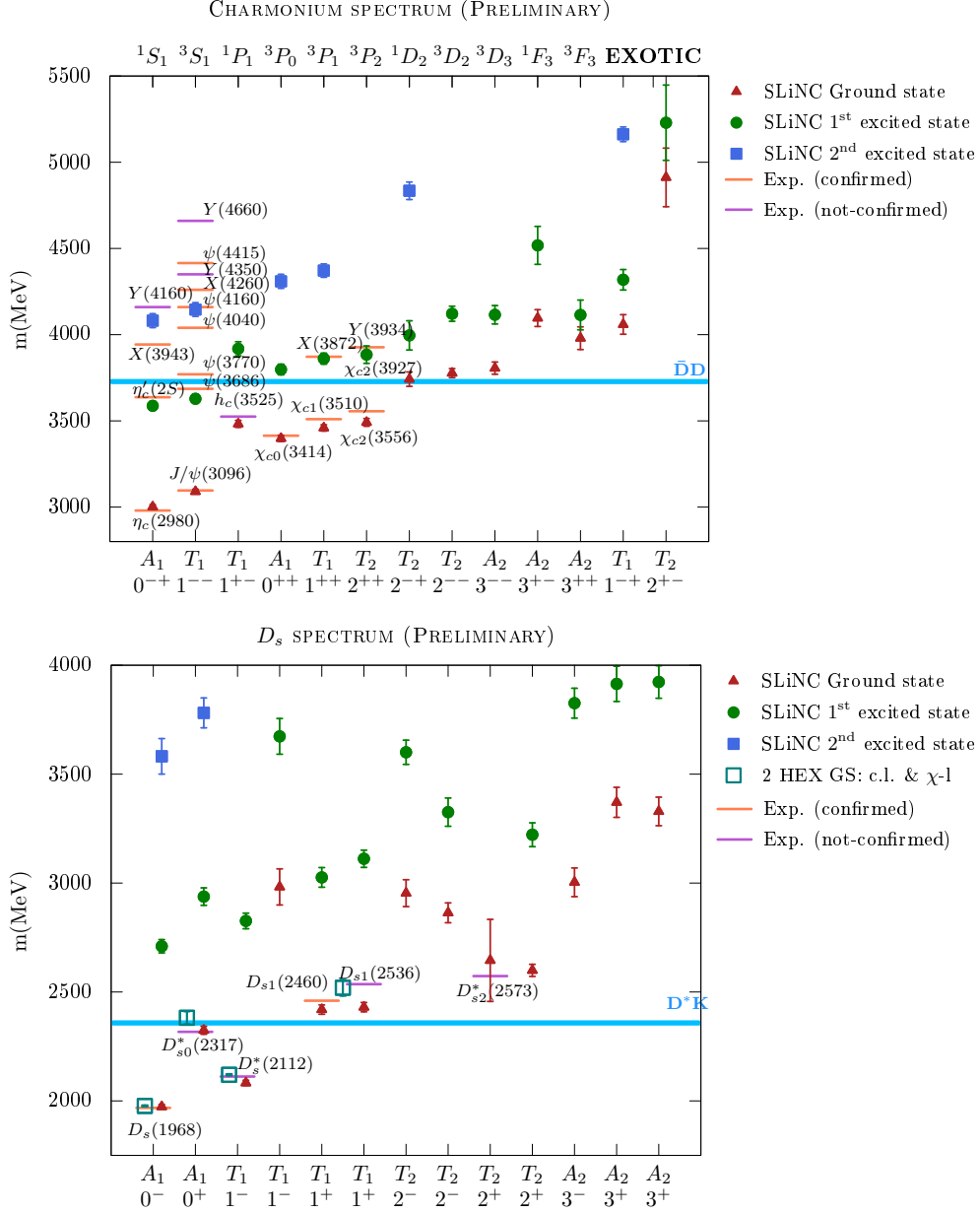
$$y^{\text{FIT}}(a, M_\pi, m_{\eta_s} | \mathbf{A}) = (1 + A_1 a^2) \left( 1 + A_2 \frac{M_\pi^2 - (M_\pi^{\text{exp}})^2}{(M_\pi^{\text{exp}})^2} + A_3 \frac{m_{\eta_s}^2 - (m_{\eta_s}^{\text{exp}})^2}{(m_{\eta_s}^{\text{exp}})^2} \right), \quad (3.1)$$

where  $y$  represents the hyperfine splitting and  $m_{\eta_s}^2 = 2m_K^2 - m_\pi^2$ . Note that, to demonstrate the quality of the fit, the data points in figure 2 are shifted to their fitted physical light quark mass values:

$$y_i^{\text{PLOT}} = y_i^{\text{RAW}} - y_i^{\text{FIT}}(a, M_\pi, m_{\eta_s} | \mathbf{A}_{\text{fitted}}) + y^{\text{FIT}}(a, 0, 0 | \mathbf{A}_{\text{fitted}}). \quad (3.2)$$

This is necessary as we have results at multiple strange quark masses for some  $\beta$ s. There is a significant dependence in the results on the lattice spacing. However, with values at 4 lattice spacings the extrapolation is under control and consistency is found with experiment. Only statistical errors have been considered so far - a full analysis of the systematic errors is underway.

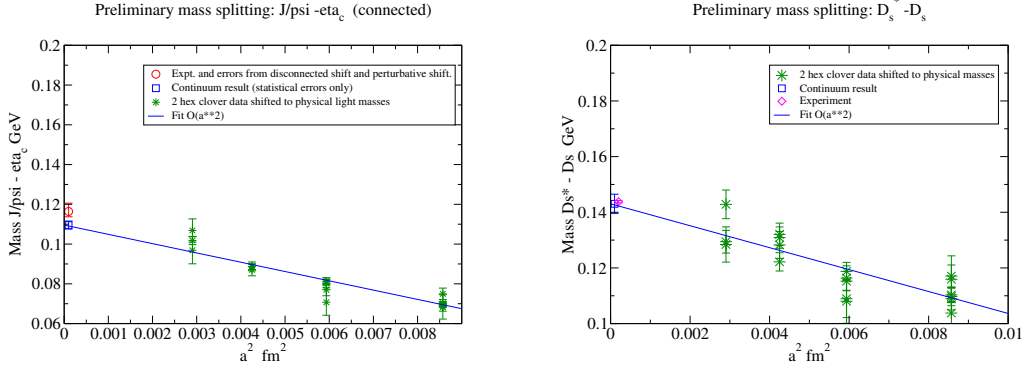
Similar results are also shown in figure 2 for the charmonium hyperfine splitting. In this case an asymmetric error bar has been added to the experimental result to reflect the fact that we have not



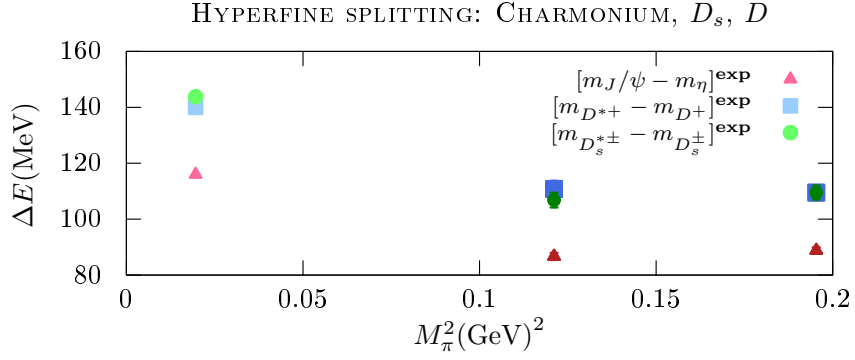
**Figure 1:** Charmonium (top) and  $D_s$  (bottom) spectra from  $N_f = 2 + 1$  configurations. The filled points are results from the QCDSF ensemble at the flavour symmetric point. The empty squares correspond to the results for the  $D_s$   $S$ -wave ground states on the BMW-c ensembles extrapolated to the continuum limit and to the physical quark masses. Only statistical errors are shown.

included the disconnected contributions. This error bar is  $+4$  and  $-2.4$  MeV based on the results of reference [15] and the estimates of [16], respectively.

Figure 3 shows the hyperfine splitting for the  $D$ ,  $D_s$  and charmonium states for the two analysed QCDSF ensembles. There is little dependence on the pion mass and the splittings are smaller than the experimental values. However, the charm quark mass needs to be tuned more precisely



**Figure 2:** Charmonium (left) and  $D_s$  (right) 1S hyperfine splittings. Asterisks (green) represent the data shifted to the fitted light quark mass values, cf. Eq. (3.2), the squares (blue) the continuum limit extrapolated results.



**Figure 3:** Hyperfine splittings (QCDSF ensembles). Triangles (dark red) correspond to the charmonium, squares (blue) to  $D$ -mesons and circles (green) to  $D_s$ -mesons. Note that for the larger pion mass the latter two data points coincide since this corresponds to the flavour symmetric point.

and, as observed in the BMW-c analysis, the continuum and quark mass extrapolation needs to be performed.

#### 4. Summary and outlook

We have presented the current status of an on-going project to study hidden and open charm meson spectra on the lattice. Results have been obtained from two sets of configurations with different actions: SLiNC, where the singlet quark mass is kept fixed and 2HEX, where the simulated pion masses range down to 120 MeV. The results presented include the charmonium and  $D_s$  spectra and the 1S hyperfine splitting. For the latter and the ground states of  $D_s$  the continuum limit and physical mass extrapolations have been performed. No other systematics nor disconnected diagrams have been included as yet.

In the near future, we expect to perform measurements for different pion masses using the QCDSF ensembles as well as for different volumes and for two different lattice spacings so that we can control the discretization errors, the finite volume effects and enable an extrapolation to the

physical quark masses. In addition, we plan to investigate the singly and doubly charmed baryons. The necessary methods, including all-to-all propagator techniques [17], have been developed to quantify mixing in the charmonium sector [18] between states of different  $L$ , with lighter flavour-singlet states and with  $D\bar{D}$  molecular states, for  $N_f = 2$  QCDSF configurations. We plan to extend this to the  $N_f = 2 + 1$  ensembles.

For the BMW-c configurations, the study of the excited states of  $D_s$  via the variational method has started. The systematics of the continuum limit and quark mass extrapolations are going to be included.

## Acknowledgements

The numerical calculations were performed on the SGI Altix ICE machines at HLRN (Berlin-Hannover, Germany) and the BlueGene/P (JuGene) and the Nehalem Cluster (JuRoPA) of the Jülich Supercomputer Center. We have made use of the Chroma software package [19] for some of the analysis. This work was supported by the EU ITN STRONGnet, the I3 HadronPhysics2 and the DFG SFB/Transregio 55. Sara Collins acknowledges support from the Claussen-Simon-Foundation (Stifterverband für die Deutsche Wissenschaft).

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